

Compiling Simple Assignments

Moo y Sagiv
Tel Aviv University
sagiv@math.tau.ac.il
and
Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-sb.de

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Subjects

- Compile vs Run Time information
- L values vs R values
- The P machine
- Code generation for expressions and assignments

Compile- vs. Run-Time Information

compile-time-information (static): information contained in or derivable from the source program

- The source code
- The types of variables (in most languages)
- The scope of variables
- The values of constants
- The addresses of static variables
- The relative addresses of automatic variables
- The size of static data (scalars, static arrays, records)
-

run-time-information (dynamic): information only available at run time

- The values of variables
- The values of conditions
- The depth of recursion
- The size of dynamic data (dynamic arrays, lists, trees)
-

L-values vs. R-values

- Assignment $x := exp$ is compiled into:
 - 1 Compute the **address** of x
 - 2 Compute the **value** of exp
 - 3 Store the value of exp at the address of x
- Generalization

R-value

$$rva(x) = \text{value of } x$$

$$rva(5) = 5$$

$$rva(x + y) = rva(x) + rva(y)$$

L-value

$$va(x) = \text{address of } x$$

$$va(5) = \text{undefined}$$

$$va(x + y) = \text{undefined}$$

$$va(a[i]) = va(a) + \text{some function of } rva(i)$$

The P-Machine



- Memory
- Instructions operate on the stack
- Typed instructions (using Pascal ordinal types)
 - +_{*i*} 1 adds the two top most int values on the stack;
 - 2 removes these from the stack;
 - 3 stores (pushes) the result on the stack
- Operand types in the P machine
 - i integer
 - r real
 - a address
 - b boolean
- Type indications in the instruction definitions
 - T a types
 - N numerical types

P-machine main loop

while true do begin

PC := *PC* + 1 ;

execute instruction in location *CODE*[*PC* - 1]

end;

Why PC increment before instruction execution?

Reason: jumps and procedure calls

Example Program

Pascal-program

```
program foo(input, output) ;  
var x, y : integer;  
begin  
    x := 3 ;  
    y := x + 7  
end.
```

P-code-program

p		initialization code
ssp	8	allocate stack space
sep	3	space for intermediate computations
ldc	a 5	pushes the address of x
ldc	i 3	pushes 3
sto	i	stores 3 in x
ldc	a 6	pushes the address of y
ldo	i 5	pushes the value of x
ldc	i 7	pushes 7
add	i	pushes x + 7
sto	i	stores x+7 in y
stp		

The definition of P-instructions

Instruction	Meaning	Condition	Result
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Instruction: name of the instruction and list of its parameters,

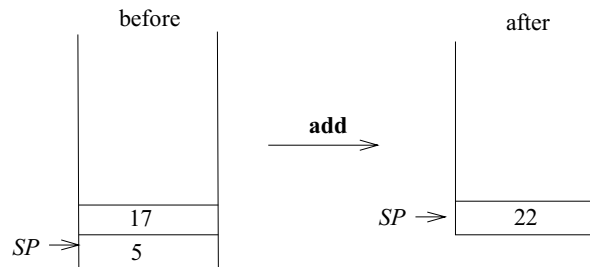
Meaning: program in a very reduced imperative language consisting of assignments between machine resources and conditions,

Condition: condition on the execution of the instruction, often a pattern describing the expected contents of the top end of the stack, i.e., the types of the cell contents,

Result: description of the result, a pattern describing the resulting stack contents

P-instructions for Arithmetic

Instr.	Meaning	Cond.	Res.
add N	$STORE[SP - 1] := STORE[SP - 1] +_N STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} N \\ N \end{pmatrix}$	(N)
sub N	$STORE[SP - 1] := STORE[SP - 1] -_N STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} N \\ N \end{pmatrix}$	(N)
mul N	$STORE[SP - 1] := STORE[SP - 1] *_N STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} N \\ N \end{pmatrix}$	(N)
div N	$STORE[SP - 1] := STORE[SP - 1] /_N STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} N \\ N \end{pmatrix}$	(N)
neg N	$STORE[SP] := -_N STORE[SP]$	(N)	(N)



P-instructions for Boolean Operations

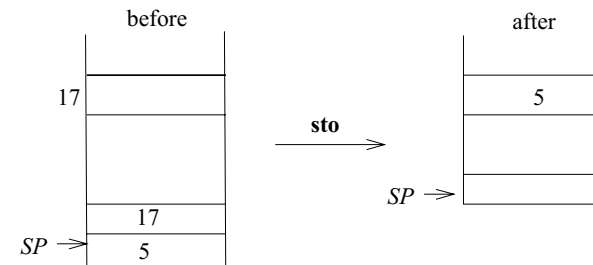
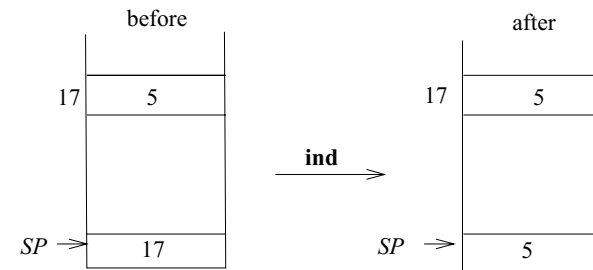
Instr.	Meaning	Cond.	Res.
and	$STORE[SP - 1] := STORE[SP - 1] \text{ and } STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} b \\ b \end{pmatrix}$	(b)
or	$STORE[SP - 1] := STORE[SP - 1] \text{ or } STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} b \\ b \end{pmatrix}$	(b)
not	$STORE[SP] := \text{not } STORE[SP]$	(b)	(b)

P-instructions for comparisons

Instr.	Meaning	Cond.	Res.
equ T	$STORE[SP - 1] := STORE[SP - 1] =_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)
geq T	$STORE[SP - 1] := STORE[SP - 1] \geq_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)
leq T	$STORE[SP - 1] := STORE[SP - 1] \leq_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)
les T	$STORE[SP - 1] := STORE[SP - 1] <_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)
grt T	$STORE[SP - 1] := STORE[SP - 1] >_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)
neq T	$STORE[SP - 1] := STORE[SP - 1] \neq_T STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T \\ T \end{pmatrix}$	(b)

P-instructions for load/store

Instr.	Meaning	Cond.	Res.
ldo Tq	$SP := SP + 1;$ $STORE[SP] := STORE[q]$	$q \in [0, maxstr]$	(T)
ldc Tq	$SP := SP + 1;$ $STORE[SP] := q$	$Typ(q) = T$	(T)
ind T	$STORE[SP] := STORE[STORE[SP]]$	(a)	(T)
sro Tq	$STORE[q] := STORE[SP];$ $SP := SP - 1$	(T) $q \in [0, maxstr]$	
sto T	$STORE[STORE[SP - 1]] := STORE[SP];$ $SP := SP - 2$	$\begin{pmatrix} a \\ T \end{pmatrix}$	



Code Generation

- Assumptions about the input program:
 - No errors (syntax, types)
 - Structure and type information is available
 - No procedures (for now)
- $\rho(v)$ is the relative address of program variable v
- invariant I_0 about the state of the P machine:

Let $code_R e \rho = is$,
 let sp be the value of SP before the execution of is
 The execution of is will leave the value of e in $STORE[sp + 1]$, and SP 's value will be $sp + 1$
 $STORE$ is otherwise unchanged

The translation of assignments and expressions

Function	Condition
$code_R(e_1 = e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{equ} T$	$Typ(e_1) = Typ(e_2) = T$
$code_R(e_1 \neq e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{neq} T$	$Typ(e_1) = Typ(e_2) = T$
$code_R(e_1 + e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{add} N$	$Typ(e_1) = Typ(e_2) = N$
$code_R(e_1 - e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{sub} N$	$Typ(e_1) = Typ(e_2) = N$
$code_R(e_1 * e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{mul} N$	$Typ(e_1) = Typ(e_2) = N$
$code_R(e_1 / e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{div} N$	$Typ(e_1) = Typ(e_2) = N$
$code_R(-e) \rho = code_R e \rho; \mathbf{neg} N$	$Typ(e) = N$
$code_R x \rho = code_L x \rho; \mathbf{ind} T$	x is a variable of type T
$code_R c \rho = \mathbf{ldc} T c$	c is a constant of type T
$code(x := e) \rho = code_L x \rho; code_R e \rho; \mathbf{sto} T$	$Typ(e) = T,$ x is a variable
$code_L x \rho = \mathbf{ldc} a \rho(x)$	x is a variable

Correctness of the code generation scheme

Proof by induction of the invariant I_0

Base cases:

$code_R x \rho$ and $code_R c \rho$

Inductive step:

Example: $code_R(e_1+e_2) \rho = code_R e_1 \rho; code_R e_2 \rho; \mathbf{add} =$
 is

Let sp be the value of SP before the execution of
 is

and Assumptions:

- The execution of $code_R e_1 \rho$ leaves the value of e_1 in $STORE[SP + 1]$ and SP has value $sp + 1$
- The execution of $code_R e_2 \rho$ leaves the value of e_2 in $STORE[SP + 2]$ and SP has value $sp + 2$

Step:

- **add** adds the topmost values and leaves the result, i.e. the value of e in $STORE[SP + 1]$ and SP has value $sp + 1$

Example

Assume that $Typ(x) = int$ and $\rho(x) = 5$

$code(x := 3) \rho$
 $= code_L x \rho; code_R 3 \rho; \mathbf{sto i}$
 $= \mathbf{ldc a 5}; code_R(3) \rho; \mathbf{sto i}$
 $= \mathbf{ldc a 5}; \mathbf{ldc i 3}; \mathbf{sto i}$

Example 2.1

Assume that $\rho(a) = 5$, $\rho(b) = 6$, and $\rho(c) = 7$ and that

$Typ(a) = Typ(b) = Typ(c) = Typ(b*c) = Typ(b+b*c) = int$

```
code(a := (b + (b * c)))  $\rho$ 
= codeL a  $\rho$ ; codeR (b + (b * c))  $\rho$ ; sto i
= ldc a 5; codeR(b + (b * c))  $\rho$ ; sto i
= ldc a 5; codeR(b)  $\rho$ ; codeR(b * c)  $\rho$ ; add i; sto i
= ldc a 5; codeL(b)  $\rho$ ; ind i; codeR(b * c)  $\rho$ ; add i; sto i
= ldc a 5; ldc a 6; ind i; codeR(b * c)  $\rho$ ; add i; sto i
= ldc a 5; ldc a 6; ind i; codeR(b)  $\rho$ ; codeR(c)  $\rho$ ; mul i; add i;
sto i
= ldc a 5; ldc a 6; ind i; codeL(b)  $\rho$ ; ind i; codeR(c)  $\rho$ ; mul i; add i;
sto i
= ldc a 5; ldc a 6; ind i; ldc a 6; ind i; codeR(c)  $\rho$ ; mul i; add i;
sto i
= ldc a 5; ldc a 6; ind i; ldc a 6; ind i; codeL(c)  $\rho$ ; ind i; mul i; add i;
sto i
= ldc a 5; ldc a 6; ind i; ldc a 6; ind i; ldc a 7; ind i; mul i; add i;
sto i
```

Interpretation of the generated code

```
q
ssp 9
sep 4
ldc a 5
ldc a 6
ind i
ldc a 6
ind i
ldc a 7
ind i
mul i
add i
sto i
```

(Non-)Optimality of the generated code

- Example 1: $a := b$
 - Generated code:
ldc a $\rho(a)$; ldc a $\rho(b)$; ind i; sto
 - Minimal code
ldo i $\rho(b)$; sro i $\rho(a)$
- Example 2: $a := a * a$;
 - Generated code:
**ldc a $\rho(a)$
ldc a $\rho(a)$
ind i
ldc a $\rho(a)$
ind i
mul i
sto**
 - Minimal code
**ldo i $\rho(a)$
dpl i
mul i
sro i $\rho(a)$**

Summary

- An inductive definition of the generated code
- Assuming that the relative addresses of variables are known at compile time
- The stack machine simplifies the task of handling complex expressions

Compiling Control Flow Statements

Moo y Sagiv
Tel Aviv University
sagiv@math.tau.ac.il

and
Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-sb.de

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Subjects

- Syntax of control flow statements
- Sequence of statements
- Conditional Statements
- Case statements
- Iterative Statements

Syntax

Sequence of Statements $st_1; st_2; \dots; st_n$

Conditional Statements

- **if** e **then** st **fi**
 - **if** e **then** st **else** st **fi**
 - **case** e **of**
 - 0: st_1 ;
 - 1: st_2 ;

 - k : st_k
- end**

Iterative Statements

- **while** e **do** st **od**
- **repeat** st **until** e

Sequence of statements

- $code(st_1; st_2)\rho = code(st_1)\rho; code(st_2)\rho$
control drops through from the last instruction generated for st_1 to the first instruction generated for st_2
 st_1 and st_2 are compiled in the same ρ

- Example: Assume that $Typ(x) = Typ(y) = int$

$$\begin{aligned} &code(x := 3; y := x + 1) \rho \\ &= code(x := 3) \rho; code(y := x + 1) \rho \\ &= code_L x \rho; code_R 3 \rho; \mathbf{sto\ i}; code(y := x + 1) \rho \\ &= \mathbf{ldc\ a} \rho(x); code_R(3) \rho; \mathbf{sto\ i} code(y := x + 1) \rho \\ &= \mathbf{ldc\ a} \rho(x); \mathbf{ldc\ i\ 3}; \mathbf{sto\ i} code(y := x + 1) \rho \\ &= \mathbf{ldc\ a} \rho(x); \mathbf{ldc\ i\ 3}; \mathbf{sto\ i} code_L y \rho; code_R(x + 1) \rho; \mathbf{sto\ i} \\ &= \mathbf{ldc\ a} \rho(x); \mathbf{ldc\ i\ 3}; \mathbf{sto\ i} \mathbf{ldc\ a} \rho(y); code_R(x + 1) \rho; \mathbf{sto\ i} \end{aligned}$$

Conditional Statements

Machine resources to implement control statements:
unconditional and conditional jumps

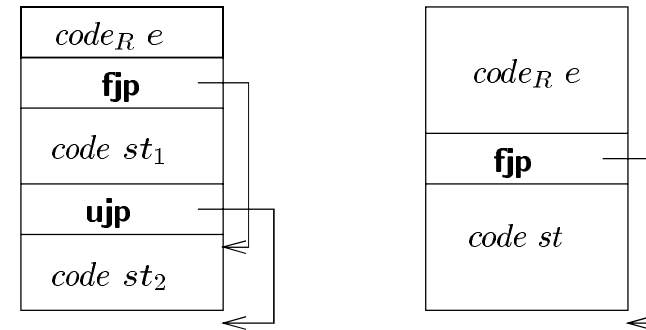
P-code Instructions for Branches

Inst.	Meaning	Cond.	Result
ujp q	$PC := q$	$q \in [0, codemax]$	
fjp q	if $STORE[SP] = false$ then $PC := q$ fi ; $SP := SP - 1$	(b) $q \in [0, codemax]$	()

fjp consumes the boolean value on top of the stack

Conditional Statements

- $code(\mathbf{if} \ e \ \mathbf{then} \ st \ \mathbf{fi}) \ \rho = code_R \ e \ \rho; \ \mathbf{fjp} \ l; \ code \ st \ \rho; \ l:$
- $code(\mathbf{if} \ e \ \mathbf{then} \ st_1 \ \mathbf{else} \ st_2 \ \mathbf{fi}) \ \rho = code_R \ e \ \rho; \ \mathbf{fjp} \ l_1; \ code \ st_1 \ \rho; \ \mathbf{ujp} \ l_2; \ l_1 : code \ st_2 \ \rho; \ l_2:$



Conditionals

doublesided

onesided

Problem: Forward references to labels

Solutions:

- Generate symbolic labels in assembly code,
- 2 pass code generation,
- “Back patching” target addresses when known

Examples

Example 1:

```
if  $a > b$  then  $c := a$  else  $c := b$  fi
```

Example 2:

```
if  $x < 5$  then if  $y < 7$  then  $c := a$  fi fi
```

Case Statement

Restricted form: finite section of the natural numbers

Input

```
case  $e$  of
```

```
  0:  $st_1$  ;
```

```
  1:  $st_2$  ;
```

```
   $k$  :  $st_k$ 
```

```
end
```

Two Solutions

- Generate nested conditionals

```
if  $e = 0$  then  $st_1$ 
```

```
else if  $e = 1$  then  $st_2$ 
```

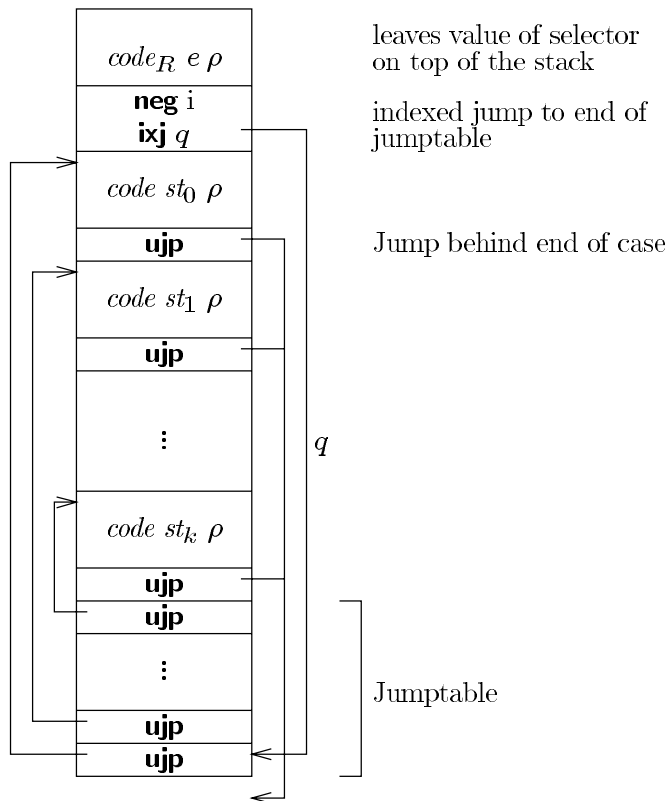
```
else if  $e = n$  then  $st_k$ 
```

```
else run time error: unmatched case label fi fi ... fi
```

- Jump table: $PC := table(e)$

P-Code Instructions for Jump Table

Instr.	Meaning	Cond.	Result
ixj q	$PC := STORE[SP] + q;$ $SP := SP - 1$	(i)	$()$



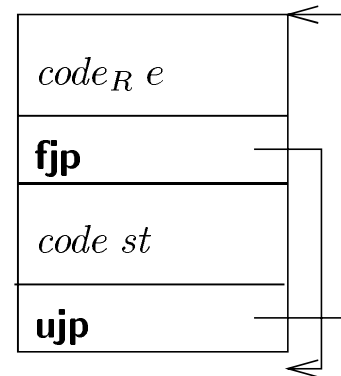
Iterative Statements

- $code$ (**while** e **do** st **od**) $\rho =$
 $l_1 : code_R e \rho; \mathbf{fjp} l_2; code st \rho; \mathbf{ujp} l_1; l_2 :$

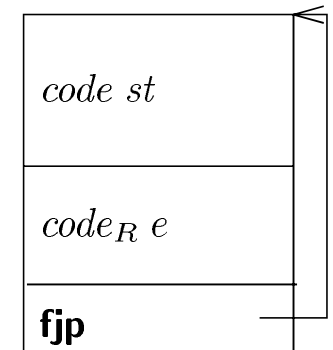
- Examp e:

$x := 1; \mathbf{while} x < 4 \mathbf{do}$
 $x := x + 1 \mathbf{od}$

- $code$ (**repeat** st **until** e) $\rho =$
 $l : code st \rho; code_R e \rho; \mathbf{fjp} l$



while e **do** st **od**



repeat st **until** e

Memory Allocation

Mooly Sagiv
Tel Aviv University
sagiv@math.tau.ac.il
and
Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-sb.de

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Subjects

- Fixed size data
- Static arrays
- Dynamic Arrays
- Records
- Pointers

Compile-Time Information on Variables

- Name
- Type
- Dimension of arrays
- Selectors in records
- Scope — when is it recognized
- Size — How many bytes are required at run-time to represent the object

Assumptions

- Code generation for a single procedure
- The first 5 stack locations are reserved; they will later be needed for procedure organization.

The function ρ

- For every data type t
 $size(t)$ is the number of bytes required at run-time to represent objects of (static) type t
- Assumption:
 $size(integer) = size(real) = size(char) = size(boolean) = 1$
- Memory allocation for variables in order of appearance
var $v_0 : t_0; v_1 : t_2; \dots; v_k : t_k;$

$$\rho(v_i) = 5 + \sum_{j=0}^{i-1} size(t_j)$$

- Example: **var** $x, y : real; z : boolean$

$$\rho = [x \mapsto 5, y \mapsto 6, z \mapsto 7]$$

Static Arrays

- The function $size$ is defined inductively
- Example:
var $a : \text{array} [-1..5, 6..7, 3..8] \text{ of integer}$
 - consists of $5 - (-1) + 1 = 7$ subarrays of type **array** $[6..7, 3..8] \text{ of integer}$
 - which consist of $7 - 6 + 1 = 2$ subarrays of type **array** $[3..8] \text{ of integer}$
 - whose sizes are $(8 - 3 + 1) \times size(int) = 6$.

Thus $size(a) = 7 * 2 * 6$ words.

- For an array type

$at : \text{array}[l_1..u_1, l_2..u_2, \dots, l_k..u_k] \text{ of } t$

$size(at) = (u_1 - l_1 + 1) * size(\text{array}[l_2..u_2, \dots, l_k..u_k] \text{ of } t)$

Therefore

$$size(at) = \prod_{j=1}^k d_j * size(t) \text{ where}$$
$$d_j = u_j - l_j + 1$$

Row Major Array Ordering

var *a*: **array**[-5..5, 1..9] **of** integer

a[-5, 1], *a*[-5, 2], ... , *a*[-5, 9],
a[-4, 1], *a*[-4, 2], ... , *a*[-4, 9],
⋮
a[5, 1], *a*[5, 2], ... , *a*[5, 9],

l-val(*a*[*i*, *j*]) = **l-val**(*a*)
+ pos. in 1st dimen. * size of 1-dim. subarray
+ pos. in 2nd dimen. * size of int
= **l-val**(*a*)
+ (*i* - (-5)) * (9 - 1 + 1)
+ *j* - 1
= **l-val**(*a*)
+ (*i* + 5) * 9 + *j* - 1
= **l-val**(*a*)
+ 9 * *i* + *j* + 44

Row Major General Array Ordering

var *a*: **array**[*l*₁..*u*₁, *l*₂..*u*₂, ..., *l*_{*n*}..*u*_{*n*}] **of** *t*

a[*l*₁, ..., *l*_{*n*}], ... , *a*[*l*₁, ..., *l*_{*n*-1}, *u*_{*n*}],
⋮
a[*u*₁, ..., *u*_{*n*-1}, *l*_{*n*}], ... , *a*[*u*₁, ..., *u*_{*n*-1}, *u*_{*n*}]

Indexing Static Arrays

Colors: **dynamic** – **static**

l-val($a[\hat{i}_1, \dots, \hat{i}_n]$)

$$\begin{aligned}
 &= \mathbf{l-val}(a) + \\
 &\quad (\hat{i}_1 - l_1) * \mathit{size}(\mathbf{array}[l_2..u_2, \dots, l_n..u_n] \text{ of } t) + \\
 &\quad (\hat{i}_2 - l_2) * \mathit{size}(\mathbf{array}[l_3..u_3, \dots, l_n..u_n] \text{ of } t) + \dots + \\
 &\quad (\hat{i}_n - l_n) * \mathit{size}(t) \\
 &= \mathbf{l-val}(a) + \\
 &\quad (\hat{i}_1 - l_1) * (\prod_{j=2}^n d_j) * \mathit{size}(t) + \\
 &\quad (\hat{i}_2 - l_2) * (\prod_{j=3}^n d_j) * \mathit{size}(t) + \dots + \\
 &\quad (\hat{i}_n - l_n) * \mathit{size}(t) \\
 &= \mathbf{l-val}(a) + \\
 &\quad \hat{i}_1 * (\prod_{j=2}^n d_j) * \mathit{size}(t) + \\
 &\quad \hat{i}_2 * (\prod_{j=3}^n d_j) * \mathit{size}(t) + \dots + \\
 &\quad \hat{i}_n * \mathit{size}(t) - \\
 &\quad (l_1 * \prod_{j=2}^n d_j + l_2 * \prod_{j=3}^n d_j + l_n) * \mathit{size}(t) \\
 &= \mathbf{l-val}(a) + \\
 &\quad \hat{i}_1 * g * d^{(1)} + \hat{i}_2 * g * d^{(2)} + \dots + \hat{i}_n * g - d * g
 \end{aligned}$$

where $\hat{i}_j = \mathbf{r-val}(i_j)$, $d^{(i)} = \prod_{j=i+1}^n d_j$, $g = \mathit{size}(t)$.

P-Code for Array Indexing

Instr.	Meaning	Cond.	Result
ixa q	$STORE[SP - 1] :=$ $STORE[SP - 1] +$ $STORE[SP] * q;$ $SP := SP - 1$	$\begin{pmatrix} i \\ a \end{pmatrix}$	(a)
inc Tq	$STORE[SP] :=$ $STORE[SP] + q$	$(T), Typ(q) = i$	(T)
dec Tq	$STORE[SP] :=$ $STORE[SP] - q$	$(T), Typ(q) = i$	(T)

$$\begin{aligned}
 \mathit{code}_L b[i_1, \dots, i_n] g \rho &= \mathbf{ldc} a \rho(b); \mathit{code}_I [i_1, \dots, i_n] g \rho \\
 \mathit{code}_I [i_1, \dots, i_n] g \rho &= \mathit{code}_R i_1 \rho; \mathbf{ixa} g * d^{(1)}; \\
 &\quad \mathit{code}_R i_2 \rho; \mathbf{ixa} g * d^{(2)}; \\
 &\quad \vdots \\
 &\quad \mathit{code}_R i_n \rho; \mathbf{ixa} g; \\
 &\quad \mathbf{dec} a g * d;
 \end{aligned}$$

Approaches to Array Bound Checking

- No checking at run-time (C)
- Generate a run-time check that the overall index expression is in range (PL1)
- Generate a run-time check that every array index is within range (Pascal, Java)

P-code for Array Checking

Instr.	Meaning	Cond.	Result
chk $p\ q$	if not $(p \leq STORE[SP] \leq q)$ then <i>error</i> ("value out of range") fi	(<i>i</i>)	(<i>i</i>)

New code for indexing including array bound checks

```

codeI [ $i_1, \dots, i_n$ ] desc  $\rho =$ 
codeR  $i_1\ \rho$ ; chk  $l_1\ u_1$ ; ixa  $g \cdot d^{(1)}$ ;
codeR  $i_2\ \rho$ ; chk  $l_2\ u_2$ ; ixa  $g \cdot d^{(2)}$ ;
  ⋮
codeR  $i_n\ \rho$ ; chk  $l_n\ u_n$ ; ixa  $g$ ;
dec  $a\ g \cdot d$ ;
    
```

where $desc = (g; l_1, u_1, \dots, l_n, u_n)$

This array description is made available through the symbol table, c.f. Semantic Analysis.

Example

```
var i, j: integer;  
    a: array[-5..5, 1..9] of integer
```

code(*a*[*i* + 1, *j*] := 0) $\rho =$

```
ldc a 7  
ldc a 5  
ind i  
ldc i 1  
add i  
chk -5 5  
ixa 9  
ldc a 6  
ind i  
chk 1 9  
ixa 1  
dec a -44  
ldc i 0  
sto i
```

Dynamic Arrays

- The size of the array is determined when the procedure is entered
- Compile-time information:
 - Dimension, i.e. the number of indices
 - Size of the components (if they are not and don't contain dynamic arrays)
- Run-time information:
 - Array bounds
 - Index ranges
 - Size of the array
 - Relative address where the array begins
 - Value of index expressions

Indexing Dynamic Arrays

Colors now: **known at indexing** – **known at creation**

$\text{l-val}(a[\hat{i}_1, \dots, \hat{i}_n])$

$$\begin{aligned}
 &= \text{l-val}(a) + \\
 &\quad (\hat{i}_1 - l_1) * (\prod_{j=2}^n d_j) * \text{size}(t) + \\
 &\quad (\hat{i}_2 - l_2) * (\prod_{j=3}^n d_j) * \text{size}(t) + \dots + \\
 &\quad (\hat{i}_n - l_n) * \text{size}(t) \\
 &= \text{l-val}(a) + \\
 &\quad (\dots ((\hat{i}_1 * d_2 + \hat{i}_2) * d_3 + \dots + \hat{i}_n) * d_n * \text{size}(t) - \\
 &\quad (\dots ((l_1 * d_2 + l_2) * d_3 + \dots + l_n) * d_n * \text{size}(t)) \\
 &= \text{l-val}(a) + d_a - d_c
 \end{aligned}$$

where \hat{i}_j is an abbreviation for $\text{rval}(i_j)$.
 $d_j = u_j - l_j + 1$ and d_c are computed once, upon array creation.
 d_a is computed for each array access.
 Computing d_a by a Horner scheme saves multiplications.
 Subtracting d_c from the array address yields the “adjusted address”.

Array Descriptor

0	Adjusted Address :a
1	Array size :i
2	Subtr. part :i
3	l_1 :i
4	u_1 :i
⋮	⋮
$2n + 1$	l_n :i
$2n + 2$	u_n :i
$2n + 3$	d_2 :i
⋮	⋮
$3n + 1$	d_n :i

$\text{code}_{Ld} c[i_1, \dots, i_k] \rho = \text{ldc } a \rho(c); \text{code}_{Id} [i_1, \dots, i_k] g \rho$
 $\text{code}_{Id} [i_1, \dots, i_n] g \rho =$

```

dpl i;           descriptor address
ind i;           adjusted address
ldc i 0;
codeR i1 ρ; add i; ldd 2n + 3; mul i;
codeR i2 ρ; add i; ldd 2n + 4; mul i;
...
codeR in-1 ρ; add i; ldd 3n + 1; mul i;
codeR in ρ; add i;
ixa g;
sli a
    
```

Additional P-code instructions

Instr.	Meaning	Cond.	Result
dpl T	$SP := SP + 1;$ $STORE[SP] :=$ $STORE[SP - 1]$	T	$\begin{pmatrix} T \\ T \end{pmatrix}$
idd q	$SP := SP + 1;$ $STORE[SP] :=$ $STORE[STORE[SP - 3] + q]$	$\begin{pmatrix} T_2 \\ T_1 \\ a \end{pmatrix}$	$\begin{pmatrix} i \\ T_2 \\ T_1 \\ a \end{pmatrix}$
sli T_2	$STORE[SP - 1] :=$ $STORE[SP];$ $SP := SP - 1$	$\begin{pmatrix} T_2 \\ T_1 \end{pmatrix}$	(T_1)

A Simplified Example

```

program test;
var i : integer ; /*  $\rho(i) = 5$  */
procedure p ;
var j, a[1..5, 1..i] of integer ; /*  $\rho(j) = 5, \rho(a) = 6$  */
begin a[3, j] := ... /*

```

```

    ldc a 6
    dpl i
    ind i
    ldc i 0
    ldc i 3
    add i
    idd 2 * 2 + 3
    mul i
    ldc a 5
    ind i
    add i
    ixa 1
    sli a
    ...
    sto i */

```

```

end;
begin read(i); p end.

```


Records

- Size known at compile time
- We assume “unique names”
- Example:

```
var v : record /*  $\rho(v) = 5$  */  
    a : integer ; /*  $\rho(a) = 0$  */  
    b : integer /*  $\rho(b) = 1$  */  
end
```

```
codeL(v.b) = ldc a 5 ; inc a 1
```

Pointers and dynamic memory allocation

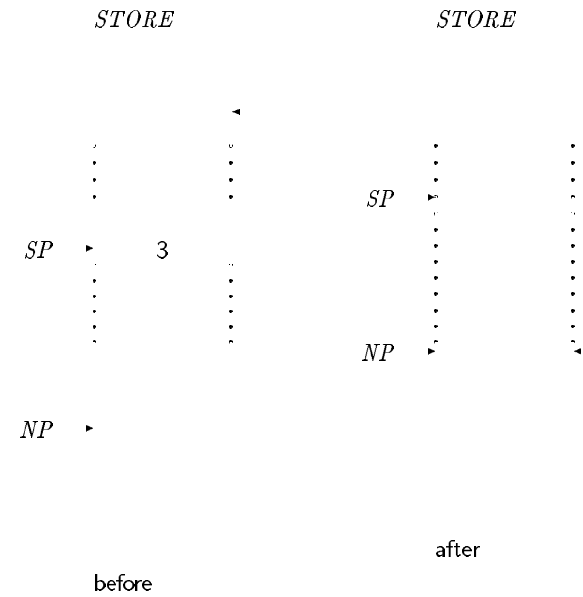
- Dynamic duration (lifetime) of the memory
- Storage can be freed in one of the following cases:
 - Can no longer be referenced (“garbage collection”)
 - Explicit deallocation (**cfree**, **dispose**)
- Space cannot be allocated on the “stack”

The Heap of the P-Machine



Instruct.	Meaning	Cond.	Result
new	if $NP - STORE[SP] \leq EP$ then <i>error</i> ("store overflow") else $NP := NP - STORE[SP];$ $STORE[STORE[SP - 1]] := NP;$ $SP := SP - 2$ fi;	$\begin{pmatrix} i \\ a \end{pmatrix}$	

The Effect of New Statement



The Generated P-Code

Assume that $Typ(x) = \uparrow t$

$code(new(x)) \rho =$
ldc a $\rho(x)$
ldc i $size(t)$
new

Complex Pointer Expressions

- Examples:
 - $x \uparrow [i + 1, j].a \uparrow [i] \uparrow$
 - $y.a.b.c \uparrow [i, j + 1].d$
- Recursive translation

$code_L(xr) \rho =$ **ldc** a $\rho(x)$; for name x
 $code_M(r) \rho =$ **inc** a $\rho(x)$; for name x
 $code_M(\uparrow r) \rho =$ **ind** a;
 $code_M([i_1, \dots, i_n]r) \rho =$ $code_{Id} [i_1, \dots, i_n] g \rho$;
 $code_M(\varepsilon) \rho =$ $code_M(r) \rho$ for array
 $= \varepsilon$

Example

```
type  t = record
      a : array[-5.. + 5, 1..9] of integer;
      b : ↑ t
    end;
var   i, j : integer;
      pt  : ↑ t;
```

$code_L\ pt \uparrow .b \uparrow .a[i + 1, j]$

ldc a $\rho(pt)$;	Load Address of pt
ind a;	Load start address of the record
inc a 99;	The start address of b
ind a;	Dereference pointer
inc a 0;	The start address of a
$code_{Id}[i + 1, j] \ 1 \ \rho$	

Handling Procedures

Mooly Sagiv
Tel Aviv University
sagiv@math.tau.ac.il
and
Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-sb.de

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Subjects

- Scoping
- Static vs. Dynamic Binding
- Calling Trees
- Static Predecessor Tree
- Parameter Passing Mechanisms
- The Run-Time Stack Frame
- Addressing of Variables
- Computing the Address Environment
- Procedures as Parameters

Procedure Incarnations

- Calling a procedure p creates an *incarnation* \hat{p}
- An incarnation of a procedure contains incarnations of the formal parameters and the local variables
- Control
 1. enters the incarnation \hat{p} from the caller \hat{q} ,
 2. may pass to a procedure incarnation \hat{r} created by a call,
 3. returns from \hat{r} ,
 4. returns from \hat{p} to \hat{q} .

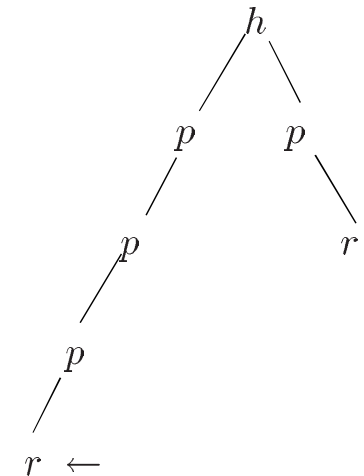
Calling Tree

- Defined for an execution sequence
- An ordered tree
- The root is the main program
- There is a node for every procedure incarnation
- If p calls p_1, p_2, \dots, p_n , then incarnations of the p_1, p_2, \dots, p_n are the children of p

A path starting with the root is called an **incarnation path**.

Example: Calling Tree

```
program  $h$ ;  
  var  $i$  : integer;  
  proc  $p$   
    proc  $r$   
      if  $i > 0$   
        then  $i := i - 1$ ;  $p$   
        else  $r$   
      fi  
     $i := 2$ ;  
     $p$ ;  
     $p$   
  end.
```



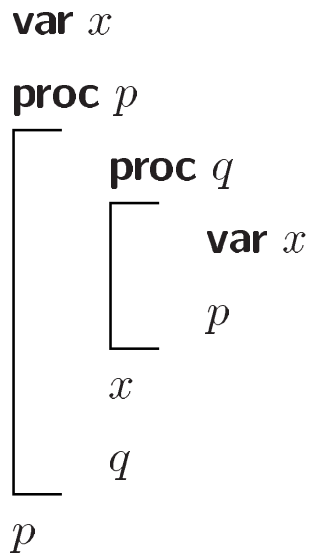
A program and its calling tree

Static vs. Dynamic Binding

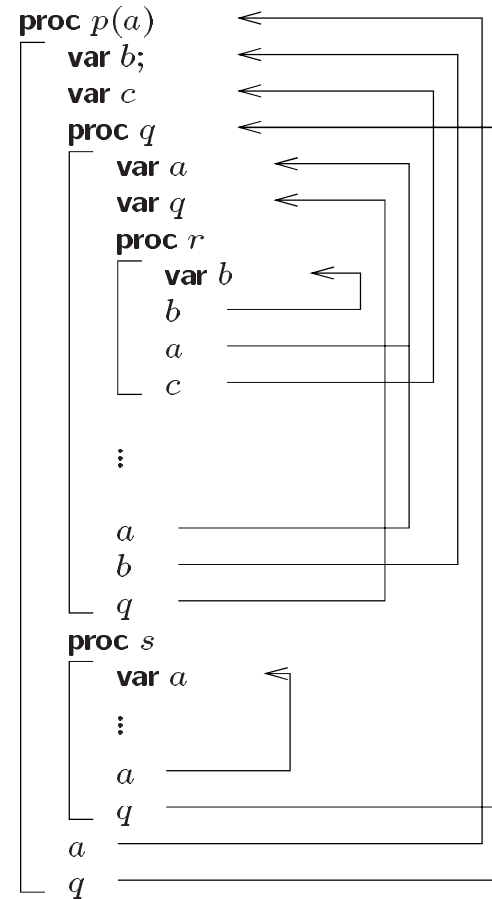
Binding strategy (scope rules) relates applied occurrences of names to defining occurrences.

Static: Applied occurrence of x denotes the defining occurrence of x in the next enclosing scope,

Dynamic: Applied occurrence of x denotes the last created incarnation of x .



Static Binding

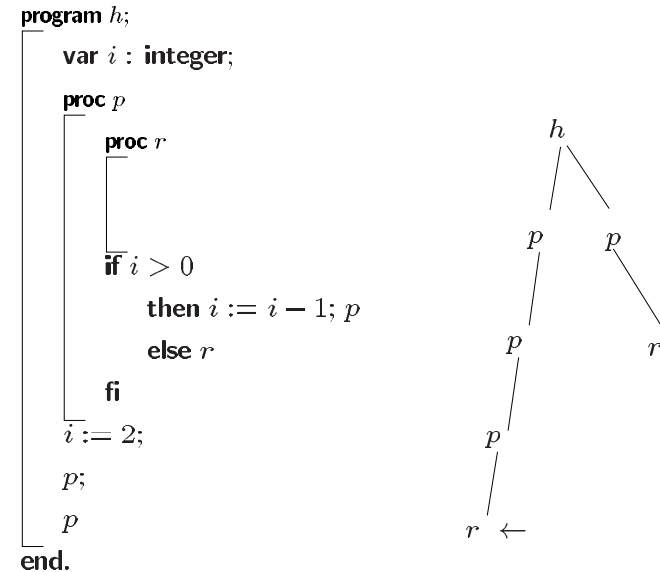


Static Predecessor Tree

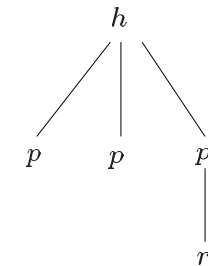
- Defined for an incarnation path of a call tree
- Its root is the main program
- There is a node for every procedure incarnation of the path
- \hat{p} is parent of \hat{q} , iff q is declared directly inside p and \hat{p} is the first incarnation of p "above" \hat{q} in the path

Warning! In general only true for programs without formal procedures.

Example: Static Predecessor Trees



A program and its calling tree



Its static predecessor tree

Parameter Passing Mechanisms

value: The r-value of the actual parameter is passed.

value-result: Only variable actual parameters allowed.
Their r-value is passed to and returned from the callee.

var/reference: Only variable actual parameters allowed.
Their l-value is passed to the callee.

name: Actual parameter is evaluated every time execution needs it.
“Thunk” (pointer to evaluation code and pointer to caller’s frame) is passed.

Example

```
program test ;
var i : integer ;
procedure p(. . . x, y: integer);
begin
    while x > 0 do
        x := x - 1 ;
        y := y + 1
    end
end
begin
    i := 3 ;
    p(i, i)
end
```

. . . = value:

. . . = var:

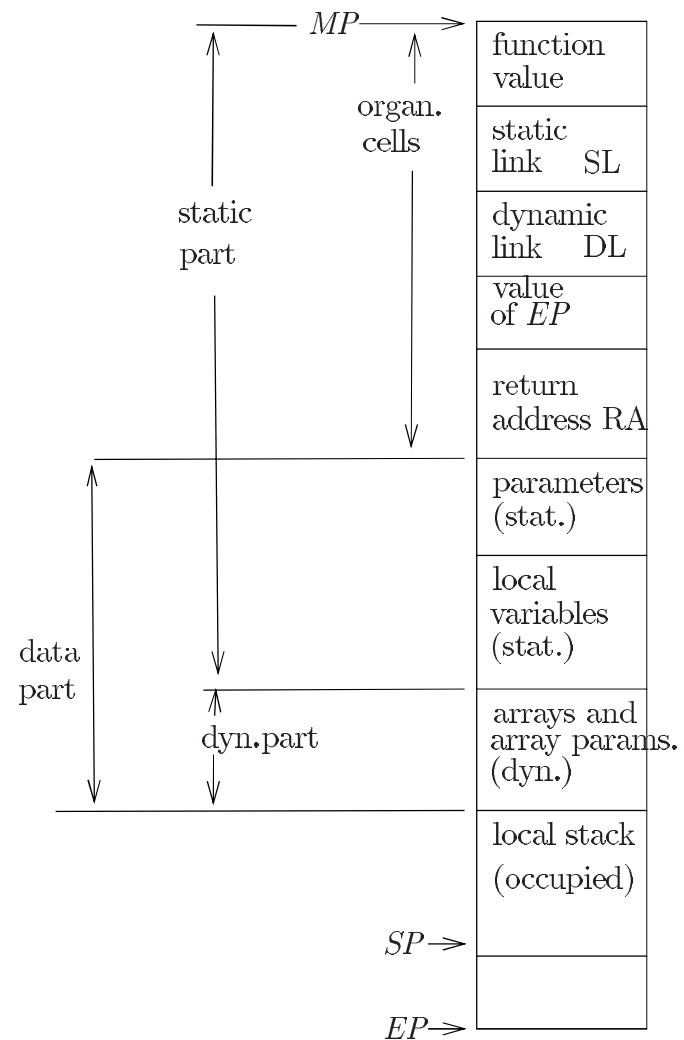
. . . = value-result:

Implementing Recursion

- Traversing the call tree with a nonrecursive program,
- needs a stack,
- elements in the stack are nodes of the call tree (i.e. procedure incarnations),
- stack content is prefix of an incarnation path (starting at the root),
- structure of this stack is isomorphic to the *run-time stack* for implementing procedures.

The Structure of an Activation Record

The representation of a procedure incarnation



Further outline

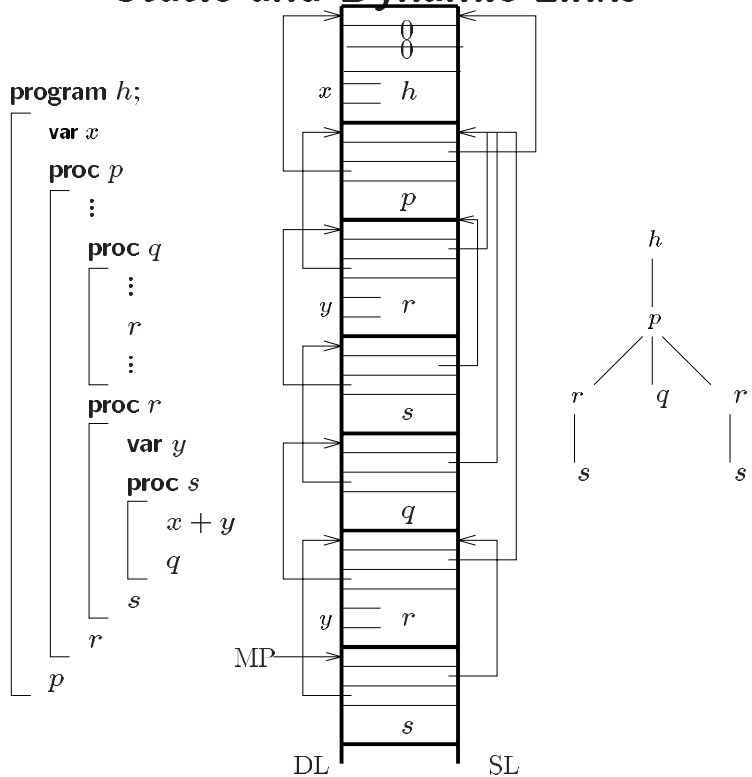
- Conceptual view of addressing
- Adapt memory allocation scheme ρ
- Adapt code for l-values
- Code for function/procedure calls/return
- Code for parameter passing
- Code for the function/procedure body

Addressing Global Variables

Using *nesting depth* of name occurrences
Inductively defined

- $nd(main) = 0$
- Types, variables, and procedures declared inside procedure p have nesting depth $nd(p) + 1$
- Applied occurrences of names inside procedure p have nesting depth $nd(p) + 1$

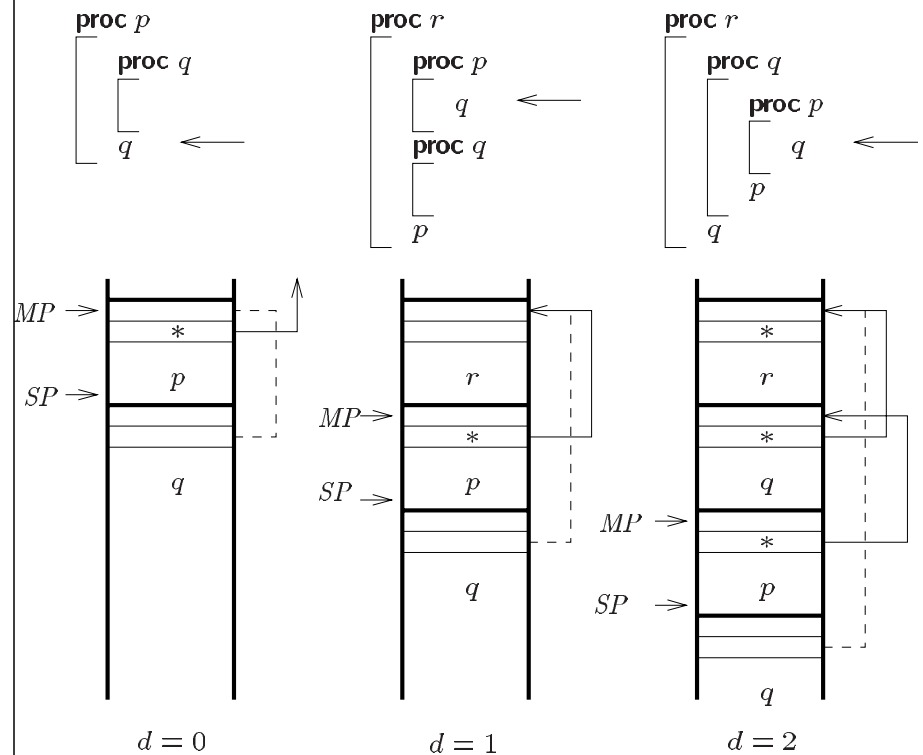
Static and Dynamic Links



program		stack configuration		corresp. tree of static pred.	
Defining	occurrence	Applied	occurrence		
<i>p</i>	1	<i>p</i>	1		
<i>q</i>	2	<i>q</i>	4		
<i>r</i>	2	<i>r</i> (in <i>p</i>)	2		
<i>s</i>	3	<i>r</i> (in <i>q</i>)	3		

Nesting depths

Updating the Static Link



Three different calls and stack configurations.

Adapt Memory Allocation Scheme

- $\rho(x) = \langle \text{relative-address}, \text{nesting depth} \rangle$
- Setting relative address remains:

$$\rho_1(v_i) = 5 + \sum_{j=0}^{i-1} \text{size}(t_j)$$

for **var** $v_0 : t_0; v_1 : t_2; \dots; v_k : t_k;$

Size Setting

Static sizes:

Parameters

var-parameters *size* = 1

value-dyn-array-parameters *size* = $3 * n + 2$

other-value-parameters *size* is unchanged

Locals

static-array *size* = $3 * n + 2 + \text{old-size}$

other-local *size* is unchanged

$3 * n + 2$ size of array descriptor

Dynamic sizes:

Dynamic arrays (locals+parameters)

as given by old *size* formula, but evaluated at run-time.

Adapt $code_L$

Instr.	Meaning
lod $T p q$	$SP := SP + 1;$ $STORE[SP] := STORE[base(p, MP) + q]$
lda $p q$	$SP := SP + 1;$ $STORE[SP] := base(p, MP) + q$
str $T p q$	$STORE[base(p, MP) + q] := STORE[SP];$ $SP := SP - 1$

$base(p, a) = \text{if } p = 0 \text{ then } a \text{ else } base(p-1, STORE[a+1])$

$code_L(x r) \rho nd = \text{lda } a d ra;$
 $code_M r \rho nd,$
 where $\rho(x) = (ra, nd')$, $d = nd - nd'$ and
 x is variable or formal value parameter

$code_L(x r) \rho nd = \text{lod } a d ra;$
 $code_M r \rho nd$
 where $\rho(x) = (ra, nd')$, $d = nd - nd'$,
 and x is formal var parameter

Code for Procedure/Function Call

1. Set static link of callee
2. Set dynamic link of callee
3. Save EP
4. Evaluate parameters (l-values, r-values)
5. Set MP
6. Save PC as return-address
7. Jump to code of callee
8. Set SP to end of static part
9. Allocate space for dynamic value arrays; copy
10. Set EP

Procedure Call

$code\ p(e_1, \dots, e_k)\ \rho\ nd =$

mst $nd - nd'$;

$code_A\ e_1\ \rho\ nd$;

⋮

$code_A\ e_k\ \rho\ nd$;

cup $s\ l$

(* $\rho(p) = (l, nd)$ *)

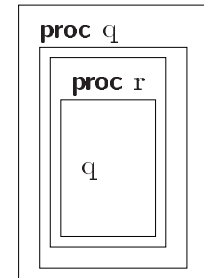
P-instructions for call and entry

Instr.	Meaning	Comment
mst p	$STORE[SP + 2] := base(p, MP);$ $STORE[SP + 3] := MP;$ $STORE[SP + 4] := EP;$ $SP := SP + 5$	Static link Dynamic link Save EP
cup $p\ q$	$MP := SP - (p + 4);$ $STORE[MP + 4] := PC;$ $PC := q$	p space for parameters Return address Jump to q
ssp p	$SP := MP + p - 1$	alloc. static area
sep p	$EP := SP + p;$ if $EP \geq NP$ then $error("store\ overflow")$ fi	alloc. temp. area Check collision stack and heap

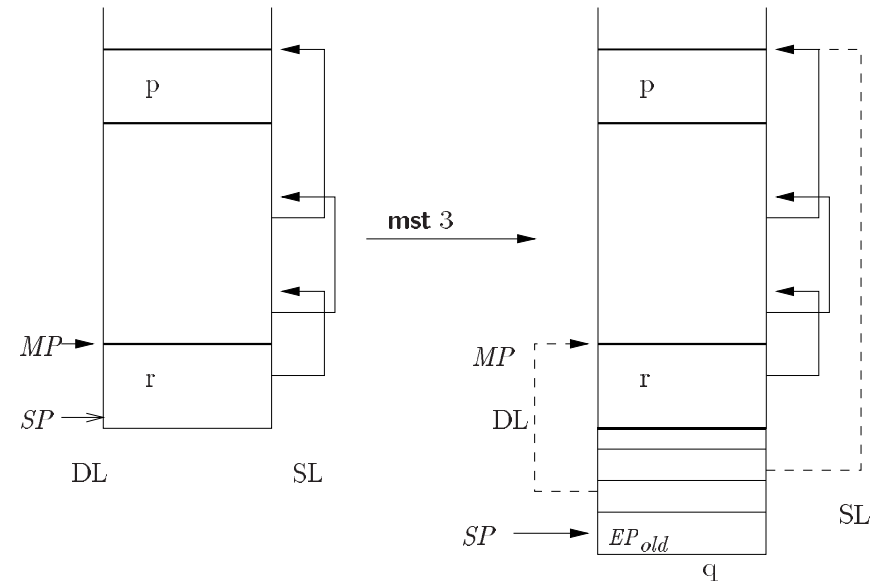
$base(p, a) = \mathbf{if}\ p = 0\ \mathbf{then}\ a\ \mathbf{else}\ base(p-1, STORE[a+1])\ \mathbf{fi}$

P-instruction mst

proc p



$nd(\text{applied occurrence of } q) -$
 $nd(\text{defining occurrence of } q) = 3$



Code for Procedure/Function Return

1. Restore SP to the beginning of current stack frame
2. Restore PC to return-address
3. Restore EP and check for heap-stack collision
4. Release frame, i.e. set MP to dynamic link

P-instructions for Return

Instr.	Meaning	Comment
retf	$SP := MP ;$ $PC := STORE[MP + 4];$ $EP := STORE[MP + 3];$ if $EP \geq NP$ then $error("store\ overflow")$ fi $MP := STORE[MP + 2]$	result is on top return address Restore EP Release frame
retp	$SP := MP - 1;$ $PC := STORE[MP + 4];$ $EP := STORE[MP + 3];$ if $EP \geq NP$ then $error("store\ overflow")$ fi $MP := STORE[MP + 2]$	No return value return address Restore EP Release frame

Procedure/Function Code

```

code (procedure p (specs); vdecls; pdecls; body) ρ nd =
    ssp n_a'';
    code_P specs ρ' nd;
    code_P vdecls ρ'' nd;
    sep k;
    ujp l;
    proc_code;                                (* local procedures *)
l : code body ρ''' nd;
retp/retf
where (ρ', n_a') = elab_specs specs ρ 5 nd
      (ρ'', n_a'') = elab_vdecls vdecls ρ' n_a' nd
      (ρ''', proc_code) = elab_pdecls pdecls ρ'' nd
    
```


elab_specs

$elab_specs : Spec^* \times Addr_Env \times Addr \times ND \rightarrow Addr_Env \times Addr$

$elab_specs (\mathbf{var} \ x : t; specs) \ \rho \ n_a \ nd =$
 $elab_specs \ specs \ \rho[(n_a, nd)/x] \ (n_a + 1) \ nd$

$elab_specs (\mathbf{value} \ x : \mathbf{array}[l_1..u_1, \dots, l_k..u_k] \ \mathbf{of} \ t'; specs) \ \rho \ n_a \ nd =$
 $elab_specs \ specs \ \rho' \ (n_a + 3k + 2) \ nd$ where
 $\rho' = \rho[(n_a, nd)/x]$
 $\quad [(n_a + 2i + 1, nd)/l_i]_{i=1}^k$
 $\quad [(n_a + 2i + 2, nd)/u_i]_{i=1}^k$

$elab_specs (\mathbf{value} \ x : t; specs) \ \rho \ n_a \ nd =$
 $elab_specs \ specs \ \rho[(n_a, nd)/x] \ (n_a + size(t)) \ nd$
for static type t

$elab_specs () \ \rho \ n_a \ nd = (\rho, n_a)$

elab_pdecls

elab_pdecls processes procedure declarations

$elab_pdecls : Pdecl^* \times Addr_Env \times ND \rightarrow Addr_Env \times Code$

$elab_pdecls (\mathbf{proc} \ p_1(\dots); \dots;$
 $\quad \vdots$
 $\quad \mathbf{proc} \ p_k(\dots); \dots;) \ \rho \ nd =$
 $(\rho', \quad l_1 : code(\mathbf{proc} \ p_1(\dots); \dots) \ \rho' \ nd + 1;$
 $\quad \vdots$
 $\quad l_k : code(\mathbf{proc} \ p_k(\dots); \dots) \ \rho' \ nd + 1)$

where $\rho' = \rho[(l_1, nd)/p_1, \dots, (l_k, nd)/p_k]$

$elab_pdecls () \ \rho \ nd = (\rho, ())$

Parameter Passing

var-actual-parameters

$$\begin{aligned} \text{code}_A x \rho nd = \\ \text{code}_L x \rho nd \end{aligned}$$

value-actual-parameters

$$\begin{aligned} \text{code}_A e \rho nd = \\ \text{code}_R e \rho nd \end{aligned}$$

value-actual-structural-parameters

$$\begin{aligned} \text{code}_A x \rho nd = \\ \text{code}_L x \rho nd; \\ \text{movs } g \end{aligned}$$

P-code for moves

Instr.	Meaning	Cond.	Res.
movs q	for $i := q - 1$ down to 0 do $STORE[SP + i] :=$ $STORE[STORE[SP] + i]$ od; $SP := SP + q - 1$	(a)	
movd q	for $i := 1$ to $STORE[MP + q + 1]$ do $STORE[SP + i] :=$ $STORE[STORE[MP + q]$ $+ STORE[MP + q + 2] + i - 1]$ od; $STORE[MP + q] :=$ $SP + 1 - STORE[MP + q + 2]$ $SP := SP + STORE[MP + q + 1]$		

Copying Dynamic Arrays

$$\begin{aligned} \text{code}_P (\text{value } x : \text{array}[u_1..o_1, \dots, u_k..o_k] \text{ of } t) \rho nd = \\ \text{movd } ra; \end{aligned}$$

The Main Program

```
code (program p (specs); vdecls; pdecls; body) ρ 0 =  
  ssp n_a;  
  code_P vdecls ρ 1;  
  sep k;  
  ujp l;  
  proc_code;  
  l : code body ρ' 1;  
  stp  
where (ρ, n_a) = elab_vdecls vdecls ∅ 5 1 and  
      (ρ', proc_code) = elab_pdecls pdecls ρ 1
```

Input Example

```
program foo ;  
var i : integer ;  
ssp 10; sep 7; ujp l1;  
function fact( n: integer): integer;  
l2: ssp 6; sep 9; ujp l3;  
begin l3 :  
  if n = 1  
    lda a 2 – 2 5; ind i; ldc i 1; equ i; fjp l4  
  then fact := 1  
  lda a 2 – 2 0; ldc i 1; sto i; ujp l5 :  
  else fact := n * fact(n – 1) fi  
  l4: lda a 2 – 2 0; lda a 2 – 2 5; ind i;  
  mst 2 – 1; lda a 2 – 2 5; ind i; ldc i 1; sub i; cup 1 l2  
  mul i; sto i; l5:  
end; retf  
begin l1 :  
  i := fact(2)  
  lda a 1 – 1 9;  
  mst 1 – 1; ldc i 2; cup 1 l2  
  sto i  
end. stp
```

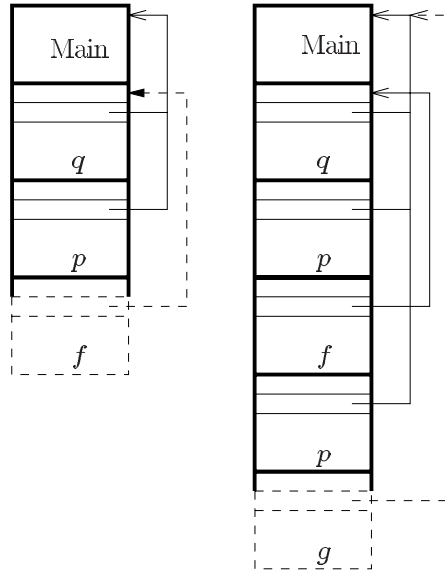
Procedures/Functions as Parameters

```

program   Main;
[
  proc p( function h)
  [
    h
  ]
  proc q
  [
    function f
    [
      p(g)
    ]
    p(f)
  ]
  function g
  [
    q
  ]
]

```

Program with formal procedures



Stacksituation after call *p*(*f*) and (dashed) after call *h*

Stacksituation after call *p*(*g*) and (dashed) after call *h*

EECS 583 – Lecture 15

Machine Information, Scheduling a Basic Block

University of Michigan

March 5, 2003

Machine Information

- ❖ Each step of code generation requires knowledge of the machine
 - » Hard code it? – used to be common practice
 - » Retargetability, then cannot
- ❖ What does the code generator need to know about the target processor?
 - » Structural information?
 - No
 - » For each opcode
 - What registers can be accessed as each of its operands
 - Other operand encoding limitations
 - » Operation latencies
 - Read inputs, write outputs
 - » Resources utilized
 - Which ones, when

Machine Description (mdes)

- ❖ Elcor mdes supports very general class of EPIC processors
 - » Probably more general than you need ☺
 - » Weakness – Does not support ISA changes like GCC
 - ❖ Terminology
 - » Generic opcode
 - Virtual opcode, machine supports k versions of it
 - ADD_W
 - » Architecture opcode or unit specific opcode or sched opcode
 - Specific assembly operation of the processor
 - ADD_W.0 = add on function unit 0
 - ❖ Each unit specific opcode has 3 properties
 - » IO format
 - » Latency
 - » Resource usage
-

- 2 -

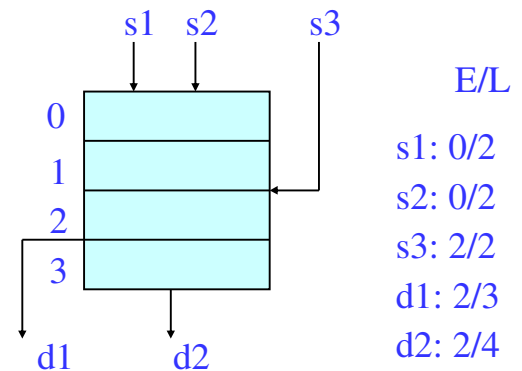
IO Format

- ❖ Registers, register files
 - » Number, width, static or rotating
 - » Read-only (hardwired 0) or read-write
 - ❖ Operation
 - » Number of source/dests
 - » Predicated or not
 - » For each source/dest/pred
 - What register file(s) can be read/written
 - Literals, if so, how big
- Multicluster machine example:
- | | |
|-----------|-------------------|
| ADD_W.0 | gpr1, gpr1 : gpr1 |
| ADD_W_L.0 | gpr1, lit6 : gpr1 |
| ADD_W.1 | gpr2, gpr2 : gpr2 |
-

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Latency Information

- ❖ Multiply takes 3 cycles
 - » No, not that simple!!!
- ❖ Differential input/output latencies
 - » Earliest read latency for each source operand
 - » Latest read latency for each source operand
 - » Earliest write latency for each destination operand
 - » Latest write latency for each destination operand
- ❖ Why all this?
 - » Unexpected events may make operands arrive late or be produced early
- ❖ Compound op: part may finish early or start late
- ❖ Instruction re-execution by
 - » Exception handlers
 - » Interrupt handlers
- ❖ Ex: `mpyadd(d1, d2, s1, s2, s3)`
 - » $d1 = s1 * s2, d2 = d1 + s3$



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Memory Serialization Latency

- ❖ Ensuring the proper ordering of dependent memory operations
- ❖ Not the memory latency
 - » But, point in the memory pipeline where 2 ops are guaranteed to be processed in sequential order
- ❖ Page fault – memory op is re-executed, so need
 - » Earliest mem serialization latency
 - » Latest mem serialization latency
- ❖ Remember
 - » Compiler will use this, so any 2 memory ops that cannot be proven independent, must be separated by mem serialization latency.

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Branch Latency

- ❖ Time relative to the initiation time of a branch at which the target of the branch is initiated
- ❖ What about branch prediction?
 - » Can reduce branch latency
 - » But, may not make it 1
- ❖ We will assume branch latency is 1 for this class (ie no delay slots!)

Example: 0: branch branch latency = k (3)
 1: xxx delay slots = k - 1 (2)
 2: yyy Note xxx and yyy are multiOps
 3: target

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Resources

- ❖ A machine resource is any aspect of the target processor for which over-subscription is possible if not explicitly managed by the compiler
 - » Scheduler must pick conflict free combinations
- ❖ 3 kinds of machine resources
 - » Hardware resources are hardware entities that would be occupied or used during the execution of an opcode
 - Integer ALUS, pipeline stages, register ports, busses, etc.
 - » Abstract resources are conceptual entities that are used to model operation conflicts or sharing constraints that do not directly correspond to any hardware resource
 - Sharing an instruction field
 - » Counted resources are identical resources such that k are required to do something
 - Any 2 input busses

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Reservation Tables

For each opcode, the resources used at each cycle relative to its initiation time are specified in the form of a table

Res1, Res2 are abstract resources to model issue constraints

relative time	Res1	Res2	ALU	MPY	Resultbus
0	X		X		
1					X

Integer add

relative time	Res1	Res2	ALU	MPY	Resultbus
0		X		X	
1				X	
2					X

Non-pipelined multiply

relative time	Res1	Res2	ALU	MPY	Resultbus
0	X	X	X		
1					X

Load, uses ALU for addr calculation, can't issue load with add or multiply

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Now, Lets Get Back to Scheduling...

❖ Scheduling constraints

- » What limits the operations that can be concurrently executed or reordered?
- » Processor resources – modeled by mdes
- » Dependences between operations
 - Data, memory, control

❖ Processor resources

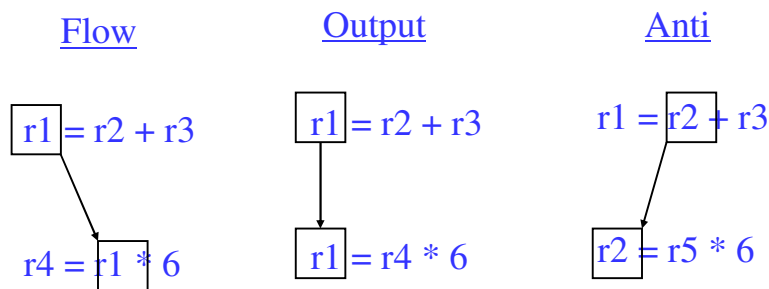
- » Manage using resource usage map (RU_map)
- » When each resource will be used by already scheduled ops
- » Considering an operation at time t
 - See if each resource in reservation table is free
- » Schedule an operation at time t
 - Update RU_map by marking resources used by op busy

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Data Dependences

❖ Data dependences

- » If 2 operations access the same register, they are dependent
- » However, only keep dependences to most recent producer/consumer as other edges are redundant
- » Types of data dependences



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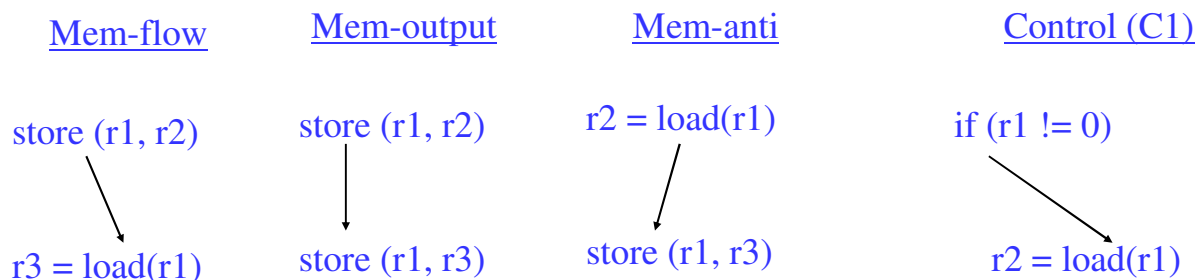
More Dependences

❖ Memory dependences

- » Similar as register, but through memory
- » Memory dependences may be certain or maybe

❖ Control dependences

- » We discussed this earlier
- » Branch determines whether an operation is executed or not
- » Operation must execute after/before a branch
- » Note, control flow (C0) is not a dependence



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Dependence Graph

- ❖ Represent dependences between operations in a block via a DAG
 - » Nodes = operations
 - » Edges = dependences
 - ❖ Single-pass traversal required to insert dependences ①
 - ❖ Example ②
 - 1: $r1 = \text{load}(r2)$
 - 2: $r2 = r1 + r4$ ③
 - 3: $\text{store}(r4, r2)$
 - 4: $p1 = \text{cmpp}(r2 < 0)$ ④
 - 5: branch if p1 to BB3 ⑤
 - 6: $\text{store}(r1, r2)$ ⑥
 - BB3: ⑥
-

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Dependence Edge Latencies

- ❖ Edge latency = minimum number of cycles necessary between initiation of the predecessor and successor in order to satisfy the dependence
 - ❖ Register flow dependence, $a \rightarrow b$
 - » $\text{Latest_write}(a) - \text{Earliest_read}(b)$
 - ❖ Register anti dependence, $a \rightarrow b$
 - » $\text{Latest_read}(a) - \text{Earliest_write}(b) + 1$
 - ❖ Register output dependence, $a \rightarrow b$
 - » $\text{Latest_write}(a) - \text{Earliest_write}(b) + 1$
 - ❖ Negative latency
 - » Possible, means successor can start before predecessor
 - » We will only deal with latency ≥ 0 , so MAX any latency with 0
-

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Dependence Edge Latencies (2)

- ❖ Memory dependences, $a \rightarrow b$ (all types, flow, anti, output)
 - » $\text{latency} = \text{latest_serialization_latency}(a) - \text{earliest_serialization_latency}(b) + 1$
 - » Prioritized memory operations
 - Hardware orders memory ops by order in MultiOp
 - Latency can be 0 with this support
- ❖ Control dependences
 - » $\text{branch} \rightarrow b$
 - Op b cannot issue until prior branch completed
 - $\text{latency} = \text{branch_latency}$
 - » $a \rightarrow \text{branch}$
 - Op a must be issued before the branch completes
 - $\text{latency} = 1 - \text{branch_latency}$ (can be negative)
 - conservative, $\text{latency} = \text{MAX}(0, 1 - \text{branch_latency})$

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Class Problem

machine model

min/max read/write latencies

```
add:  src 0/1
      dst 1/1
mpy:  src 0/2
      dst 2/3
load: src 0/0
      dst 2/2
      sync 1/1
store: src 0/0
      dst -
      sync 1/1
```

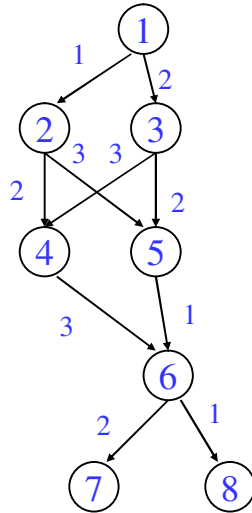
1. Draw dependence graph
2. Label edges with type and latencies

```
r1 = load(r2)
r2 = r2 + 1
store (r8, r2)
r3 = load(r2)
r4 = r1 * r3
r5 = r5 + r4
r2 = r6 + 4
store (r2, r5)
```

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Dependence Graph Properties - Estart

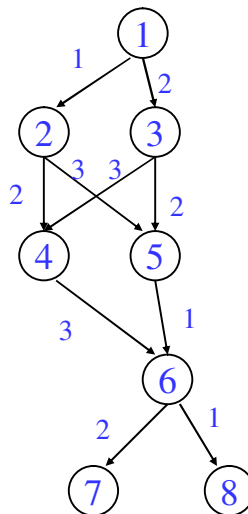
- ❖ Estart = earliest start time, (as soon as possible - ASAP)
 - » Schedule length with infinite resources (dependence height)
 - » Estart = 0 if node has no predecessors
 - » Estart = MAX(Estart(pred) + latency) for each predecessor node
 - » Example



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Lstart

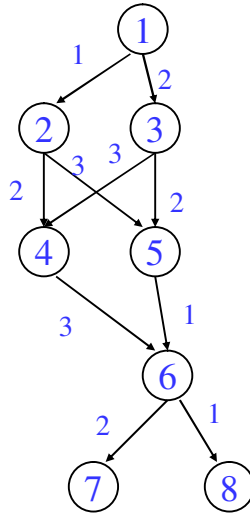
- ❖ Lstart = latest start time, ALAP
 - » Latest time a node can be scheduled s.t. sched length not increased beyond infinite resource schedule length
 - » Lstart = Estart if node has no successors
 - » Lstart = MIN(Lstart(succ) - latency) for each successor node
 - » Example



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Slack

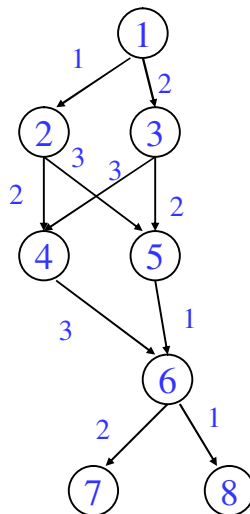
- ❖ Slack = measure of the scheduling freedom
 - » Slack = $L_{start} - E_{start}$ for each node
 - » Larger slack means more mobility
 - » Example



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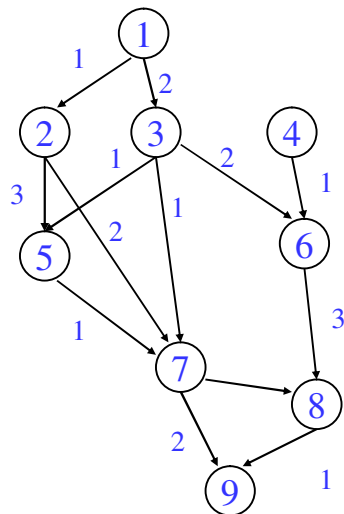
Critical Path

- ❖ Critical operations = Operations with slack = 0
 - » No mobility, cannot be delayed without extending the schedule length of the block
 - » Critical path = sequence of critical operations from node with no predecessors to exit node, can be multiple crit paths



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Class Problem



Node	Estart	Lstart	Slack
1			
2			
3			
4			
5			
6			
7			
8			
9			

Critical path(s) =

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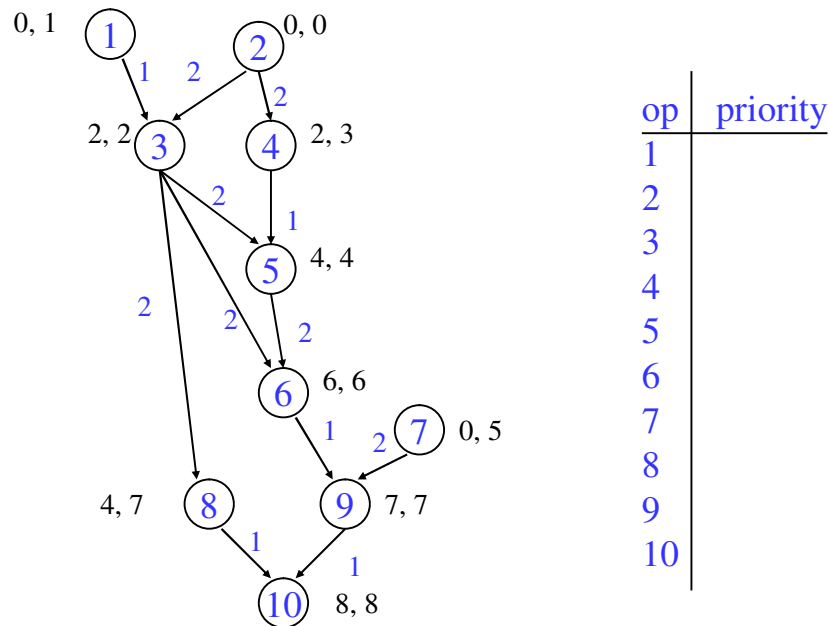
Operation Priority

- ❖ Priority – Need a mechanism to decide which ops to schedule first (when you have multiple choices)
- ❖ Common priority functions
 - » Height – Distance from exit node
 - Give priority to amount of work left to do
 - » Slackness – inversely proportional to slack
 - Give priority to ops on the critical path
 - » Register use – priority to nodes with more source operands and fewer destination operands
 - Reduces number of live registers
 - » Uncover – high priority to nodes with many children
 - Frees up more nodes
 - » Original order – when all else fails

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Height-Based Priority

- ❖ Height-based is the most common
 - » $\text{priority}(\text{op}) = \text{MaxLstart} - \text{Lstart}(\text{op}) + 1$



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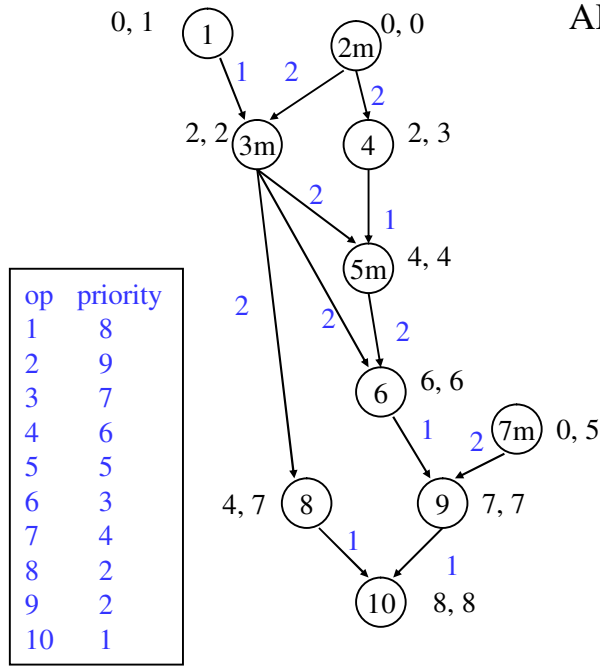
List Scheduling (Cycle Scheduler)

- ❖ Build dependence graph, calculate priority
- ❖ Add all ops to UNSCHEDULED set
- ❖ $\text{time} = -1$
- ❖ while (UNSCHEDULED is not empty)
 - » $\text{time}++$
 - » $\text{READY} = \text{UNSCHEDULED}$ ops whose incoming dependences have been satisfied
 - » Sort READY using priority function
 - » For each op in READY (highest to lowest priority)
 - op can be scheduled at current time? (are the resources free?)
 - ♦ Yes, schedule it, $\text{op.issue_time} = \text{time}$
 - ↓ Mark resources busy in RU_map relative to issue time
 - ↓ Remove op from UNSCHEDULED/READY sets
 - ♦ No, continue

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Cycle Scheduling Example

Machine: 2 issue, 1 memory port, 1 ALU
 Memory port = 2 cycles, non-pipelined
 ALU = 1 cycle

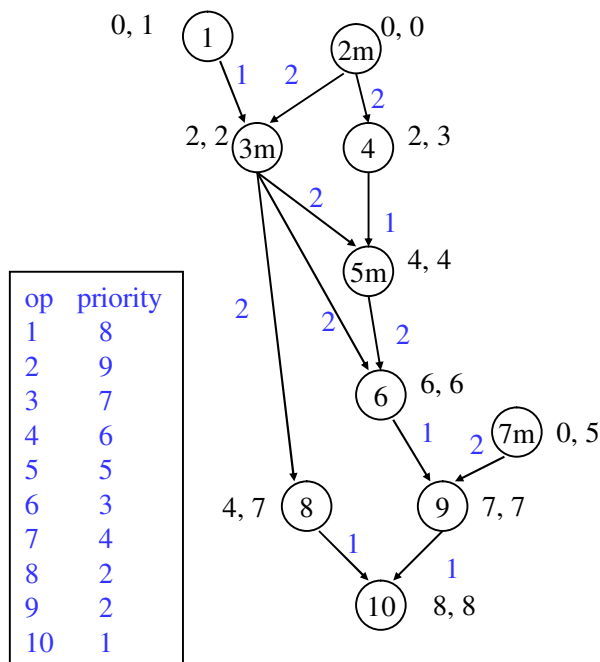


RU_map

time	ALU	MEM
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

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Cycle Scheduling Example (2)



RU_map

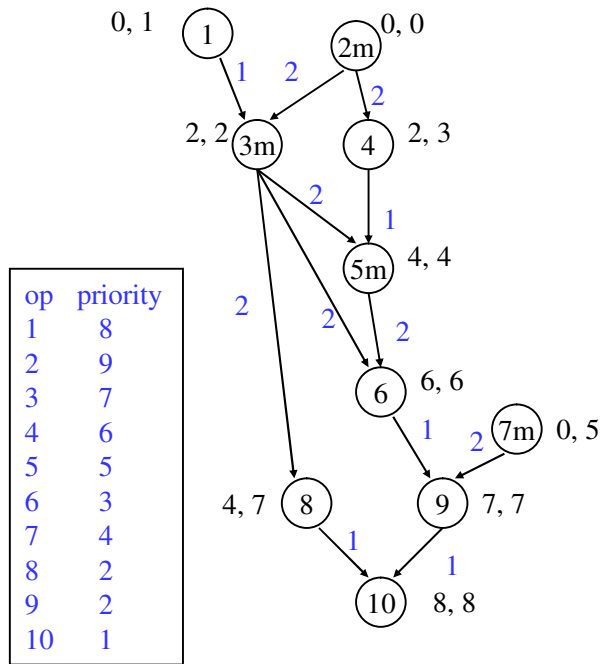
Schedule

time	ALU	MEM
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

time	Ready	Placed
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

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Cycle Scheduling Example (3)

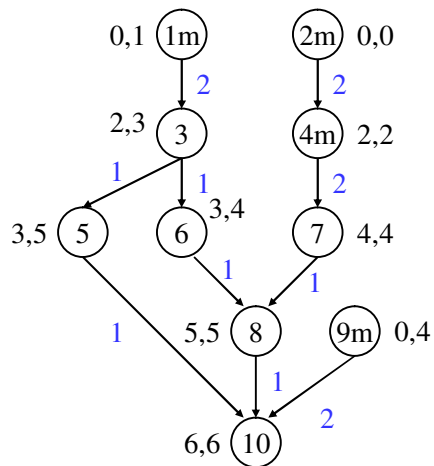


Schedule

time	Ready	Placed
0	1,2,7	1,2
1	7	-
2	3,4,7	3,4
3	7	-
4	5,7,8	5,8
5	7	-
6	6,7	6,7
7	-	-
8	9	9
9	10	10

Class Problem

Machine: 2 issue, 1 memory port, 1 ALU
 Memory port = 2 cycles, pipelined
 ALU = 1 cycle



1. Calculate height-based priorities
2. Schedule using cycle scheduler